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13. ABSTRACT (Maximum 200 words)

This final report summarizes the achievements of our research program, and describes the research results obtained during the past three years. Our work has focused on Air Force application platforms of compressor control, and design of missile autopilot. Several results were obtained in the past three years. The first one was local feedback control laws that stabilize the critical operating condition of the axial flow compression system governed by Moore-Greitzer model. The second one was missile autopilot design with time-varying control techniques where time-varying spectrum theory was used. The last one was the simulation study for feedback stabilization of compression control systems by software programming in Simulink Toolbox. In the past three years, the funding for this proposal was able to support Phillip Martin to complete his Master degree, and Michael Mickle to complete most part of his Ph.D Dissertation. Their work produced two journal publications and three conference papers. Therefore our goals for this program were mostly accomplished.

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Robust Adaptive Control
and Its Applications to Missile Autopilots

AFOSR Grant No. F49620-95-1-0354

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Final Report for

AFOSR ASSERT Program

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Robust Adaptive Control and Its Applications to Missile Autopilots

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Objectives

The primary objective of this research program was novel approaches to adaptive modeling and control for multivariable feedback systems in the face of model uncertainties and nonlinearities, as well as its applications to missile autopilots. Since its inception, this objective is altered due to mainly the change of research effort for the parent proposal to compressor control. Hence compressor control became an integral part of this program. However the application platform of missile autopilots still remained as one of the focuses in this program using time-varying control theory, in stead of adaptive control method. The new objectives are: (1) developing effective feedback control laws for suppression of rotating stall dynamics in axial flow compressors; and (2) developing new design methods based on time-varying spectrum theory for missile autopilots. The objective (1) was a result of collaboration of the PI with Dr. Siva Banda in Control and Dynamics Lab. at WPAFB (Wright-Patterson Air Force Base). The change of research objective was consulted with Dr. Marc Jacobs, AFOSR Program Manager.

1 Introduction

Our research program began in June 1, 1995. Initially, our plan was to train one Ph.D. student Brendon Randolph who is an African American in the area of robust adaptive control with missile autopilots as an application platform. However since then there had been two changes. The first was the change of interest for Mr. Brendon Randolph. Although he was enthusiastic about pursuing a Ph.D. degree initially, he lost his interest after taking a language class in Spring of 1995. He became fasinated about linguistic courses, and decided not to do Ph.D. in Electrical Engineering. The PI had to recruit new students to fulfill the proposed research plan. The PI got a new student: Phillip Martin who was a senior undergraduate student in 1995. He worked for the PI as a student

worker at the beginning year of this prohgram, and then as a research assistant after he enrolled in Graduate School at LSU. Since Fall 1997, the PI got Michael Mickle, a Ph.D. student working in the area of missile autopilot design using time-varying control method. Hence our research program was composed of two students, both U.S. citizens. The second change in our program was the research objective which was initially development of innovative robust adaptive control algorithms applied to missile autopilots. The PI worked with the control group in WPAFB as an AFOSR Summer Research Faculty Fellow in summer of 1995. As a result of collaborative work with the Dynamics and Control Laboratory of WPAFB, the parent proposal [4] changed its focus from theory oriented identification and control for linear uncertain systems to application oriented identification and control for axial flow compressors which are the essential part of aeroengines. The change of research objectives for the parent proposal influences the change of our ASSERT program. Thus compressor control became an important part of this program. The work of Phillip Martin focused on rotating stall control for improvement of compressor performance. However design of missile autopilot was not ignored. Because of the expertise of Michael Mickle, he worked as a research assistant for the PI focusing on missile autopilot design using time-varying spectrum theory.

Since the inception of the ASSERT program, significant progress has been made for compressor control, and missile autopilot design. Several control laws were developed for stabilization of rotating stall for the Moore-Greitzer model by the PI and his co-worker of which Phillip Martin made his contributions. He wrote most of the Simulink programs for numerical simulations of the Moore-Greitzer model, and participated all the research work on rotating stall control for the third order Moore-Greitzer model. In addition, he investigated effectiveness of the existing feedback control laws for axial flow compressors by other researchers in the field, and compared performance on suppression of rotating stall. These results were summarized in a technical report. For design of missile autopilots, progress was made using time-varying spectrum theory. Several results were obtained. The first was the use of PD (parallel Differential) spectral theory for autopilot design that exponentially stabilizes the linearized dynamics about the desired trajectory. The design relies on a Lyapunov transformation of the linearized dynamics which is applicable to any n-th order, uniformly completely controllable systems. The second is the use of EMA (extended-mean assignment) control technique, developed again for time-varying systems, for missile autopilot design to achieve angle of attack and nominal acceleration tracking. The sallient feature of the tracking controller is good tracking performance for arbitrary trajectory without scheduling of any constant design parameters through the entire Mach operating range. These results provide new design tools for missile autopilots and complement the existing design methods based on gain schedule control.

Regarding education, this research program has made a great effort to train graduate students.

In the past three years, the PI offered a special topics course on bifurcation analysis and compressor control. He also taught one special topics class on robust identification in $\mathcal{H}_\infty$. More than 20 students were benefited through the special topics classes on bifurcation analysis and compressor control, and on robust identification in $\mathcal{H}_\infty$. Moreover the funding of this proposal supported Phillip Martin to complete his M.S. degree in the research area of rotating stall control, and Michael Mickle to work on his dissertation which is expected to complete by December of 1998 in the area of missile autopilot design. At present, Mr. Mickle is supported by the department with teaching assistantship. Considering that our initial education plan was training one Ph.D. student and three undergraduate students in summer, it was an accomplishment for us to train one Ph.D. student, and one M.S. student (who was supported as an undergraduate student worker at the beginning).

This program would like to thank Dr. Marc Jacobs and AFOSR for giving us the opportunity to work under the Program of Dynamics and Control, AFOSR, and to contribute to Air Force missions. This final report summarizes our achievements in both research and education during the past three years, and describes in details the results obtained by our research program. Our research findings will be reported in the next section.

2 Accomplishments/New Findings

Since the inception of our program in June 1, 1995, the research effort of our program has focused on both rotating stall control for axial flow compressors, and design of missile autopilots. Axial flow compressors are the vital part of the aeroengine, and rotating stall limits effectively the aeroengine performance. The performance improvement for axial flow compressors will allow lighter jet engines for airplanes that is crucial for DoD missions. Hence the transition of the parent proposal to application oriented research on compressor control was in the interest of the Air Force, which was approved by Program Manager Dr. Marc Jacobs. Consequently rotating stall control became an essential part of our program. However research work on missile autopilot continued which produced new design methods based on time-varying thoery. These results are summarized in the next two subsections.

2.1 Stabilization of Rotating Stall Dynamics

Rotating stall is a fundamental aerodynamic instability in axial flow compressors, induced by nonlinear bifurcation. It effectively reduces the performance of aeroengines. Our work has focused on compression control systems with throttle position employed as actuator and pressure rise as output measurement. The advantage of our proposed feedback control system lies in 1-D actuator and 1-D sensor. This was different from the previous work in rotating stall control. In particular, linear

control method of Paduano *et al.* [10] employed inlet guide vanes as actuators and local gas flow rate as output measurements that required 2-D actuator and 2-D sensor. In the work of Liaw and Abed [6], nonlinear feedback control law was used with 1-D actuator. However an issue with this feedback control law was the difficulty to obtain the disturbance amplitude A, the feedback variable. For this an array of 2-D sensors had to be employed with hot wires on the circumference of the compressor in order to measure the local mass flow rate and to estimate the disturbance amplitude A, especially for high order compressor models. These are delicate devices which are unlikely to survive the hostile environment of the axial flow compressor. That is again in contrast to pressure transducers which are more reliable and durable to the volatile flow field. Thus the feedback control system proposed in our program represented a practical approach to suppressing rotating stall in axial flow compressors. However the feedback control problem with throttle position as actuator and pressure rise as output measurement also posed a challenge because the critical mode of the linearized system corresponding to rotating stall is neither controllable, nor observable. Our accomplishments or new findings for rotating stall control are the following:

(1) Local stabilization for bifurcated systems involving steady-state equilibrium whose linearized critical mode is neither controllable nor observable.

In order to tackle the rotating stall problem, a general problem was studied first: developing output feedback stabilization law for bifurcated systems whose linearized critical mode was neither controllable, nor observable. This problem was motivated by the fact that rotating stall in third order Moore-Greitzer model corresponds to pitchfork bifurcation. The hysteresis loop associated with rotating stall is due to the subcritical nature of the pitchfork bifurcation. Clearly linear theory is inadequate to solving this problem.

To be more specific, consider the following nth order parametrized nonlinear control system:

$$\dot{x} = f(\gamma, x) + g(x)u, \quad y = cx, \quad f(\gamma, x_e) \equiv 0, \tag{1}$$

where $x \in \mathbf{R}^n$, $u \in \mathbf{R}$, and γ is a real-valued parameter. The linearized system around $x_e(\gamma)$ and $u = 0$ is given by:

$$\dot{x} = L(\gamma)x + bu, \quad y = cx, \quad L(\gamma) = \left.\frac{df(\gamma, x)}{dx}\right|_{x=x_e}, \quad b = g(x_e), \quad f(\gamma, x_e) = 0. \tag{2}$$

Assume that $f(\cdot, \cdot)$ is sufficiently smooth such that the equilibrium solution x_e, when $u = 0$, is a smooth function of γ, and $L(\gamma)$ possesses a simple eigenvalue $\lambda(\gamma)$ satisfying

$$\lambda(\gamma_c) = 0, \quad \lambda'(\gamma_c) = \frac{d\lambda}{d\gamma}(\gamma_c) < 0,$$

while all other eigenvalues are stable for $\gamma \geq \gamma_c$, here γ_c is the critical value of γ. This assumption implies that the equilibrium solution $x_e(\gamma)$ of the unforced system is locally stable for all $\gamma > \gamma_c$ and becomes unstable for $\gamma < \gamma_c$. More importantly, additional equilibrium solutions will be born at $\gamma = \gamma_c$ that are assumed to be unstable for $\gamma \leq \gamma_c$. Our first task was to investigate stabilization of bifurcated solutions under the condition that the critical mode corresponding to the critical eigenvalue $\lambda(\gamma)$ is neither controllable, nor observable. Our result was the following [3].

Theorem 2.1 *Consider the nonlinear control system in (1) in its Taylor series expansion form:*

$$\dot{x} = L_0 x + \delta\gamma L_1 x + u\tilde{L}_1 x + bu + Q_0[x,x] + (\delta\gamma)^2 L_2 x + \delta\gamma Q_1[x,x] + u\tilde{Q}_1[x,x] + C_0[x,x,x] + \cdots \quad (3)$$

with output feedback control law in the form of:

$$u = K(y) = K_1 y + K_2 y^2 + K_3 y^3 + \cdots, \quad K(0) = 0, \quad y = cx, \quad (4)$$

where $\tilde{L}_1 x$ and $\tilde{Q}_1[x,x]$ are linear and quadratic components of $g(x)$. Suppose that the bifurcated solution at $\gamma = \gamma_c$ is not locally stable, and the critical mode of the linearized system at $\gamma = \gamma_c$ is neither controllable nor observable. Then (i) For the case $\delta\gamma_1 \neq 0$, i.e., $\ell Q_0[r,r] \neq 0$, there does not exist a feedback control law $u = K(y)$, $y = cx$, that stabilizes the given branch of the bifurcated solution; (ii) For the case $\delta\gamma_1 = 0$, there exists a feedback control law $u = K(y)$, $y = cx$ that stabilizes the bifurcated solutions, if and only if there exists $K_1 \neq 0$ such that the none zero eigenvalues of $L_0^* = L_0 + bK_1 c$ remain in the open left half plane and

$$\tilde{\lambda}_2 - 4\ell\left(Q_0[r, x_{e2} - x_{e2}^*]\right) + 2(\ell\tilde{L}_1 r)(K_1 c x_{e2}^*) < 0 \quad (5)$$

where K_1 is the linear gain of the feedback controller, and

$$x_{e2} = -(\ell^T \ell + L_0^T L_0)^{-1} L_0^T Q_0[r,r], \quad x_{e2}^* = -(\ell^T \ell + (L_0^*)^T L_0^*)^{-1} (L_0^*)^T Q_0[r,r]. \quad (6)$$

If the above condition holds, then the stabilizing feedback controller can be chosen to be linear.

The uncontrollability and unobservability for the critical mode imply that the high order terms determine local stability of the critical dynamics. However it is interesting to note that although the nonlinear feedback controller in (4) is employed, only the linear term has the effect on the stability of the bifurcated solution according to the second part of Theorem 2.1. Higher order terms are unnecessary if bifurcation stabilization is the sole interest. Recall that in pure linear system case, if an unstable mode is uncontrollable and unobservable, it would be impossible to design feedback controller to stabilize the system. For our problem, stabilization of bifurcated solution is achieved through influencing the quadratic term that does not exist for linear systems. Our result gave a complete solution to the bifurcation stabilization problem posed at the beginning. Moreover, our result gave an explicite formula to complete the stabilization gain, if it exists.

(2) Linear and nonlinear feedback stabilization laws for our proposed compression control system that eliminate the hysteresis loop induced by rotating stall.

The theoretical results for bifurcation stabilization was used for design of rotating stall controller with throttle as actuator and pressure rise as output measurement. The compression control system in consideration was the Moore-Greitzer model:

$$\begin{aligned}
\frac{d\Psi}{d\tau} &= \frac{1}{\beta^2}\left(\Phi - \Phi_\gamma(\Psi)\right), & \Phi_\gamma(\Psi) &= (\gamma_0 + u)\sqrt{\Psi} - 1, \ \ \gamma > 0, & (7)\\
\frac{d\Phi}{d\tau} &= -\Psi + \psi_c(\Phi) + 6c_3\Phi A^2, & \psi_c(\Phi) &= c_0 + c_1\Phi + c_3\Phi^3, & (8)\\
\frac{dA}{d\tau} &= \sigma A\left(1 - \Phi^2 - A^2\right), & \sigma &> 0, & (9)
\end{aligned}$$

where Ψ is the (converted) pressure rise, $\Phi = \frac{\bar{\phi}}{W} - 1$ where $\bar{\phi}$ is the circumferential mean of local flow coefficient and W is a constant, A is the disturbance amplitude, τ is the (converted) time variable, $\gamma = \gamma_0 + u$ is the (converted) throttle position with γ_0 the nominal position and u the actuating input, and β and σ are parameters. It is assumed that $c_0 + c_1 + c_3 > 0$, $0 < c_0 < -10c_3$, and $c_3 < 0$. A detailed bifurcation analysis of this model can be found in [8, 1, 6]. It is now well known that the rotating stall is induced by subcritical pitchfork bifurcation which is the very reason for the existence of hysteresis loop associated with rotating stall. Suppose that (Ψ_e, Φ_e, A_e) where $\Psi_e \neq 0$, $\Phi_e \neq 0$, and $A_e = 0$, is the set of the desired equilibrium point. Clearly the equilibrium point is a function of γ. Then the linearized system is given by

$$\begin{bmatrix} \dot{\Psi} \\ \dot{\Phi} \\ \dot{A} \end{bmatrix} = \begin{bmatrix} -\beta^{-2}\Phi'_\gamma(\Psi_e) & \beta^{-2} & 0 \\ -1 & \psi'_c(\Phi_e) & 0 \\ 0 & 0 & \sigma(1-\Phi_e^2) \end{bmatrix} \begin{bmatrix} \delta\Psi \\ \delta\Phi \\ A \end{bmatrix} + \begin{bmatrix} -\beta^{-2}\Phi'_\gamma(\Psi_e) \\ 0 \\ 0 \end{bmatrix} u, \quad y = \delta\Psi, \qquad (10)$$

where $\delta\Psi = \Psi - \Psi_e$, and $\delta\Phi = \Phi - \Phi_e$. It is easy to see that the critical eigenvalue is $\lambda(\gamma) = \sigma(1 - \Phi_e^2(\gamma))$ that is neither controllable, nor observable. Our goal was to synthesize a local nonlinear feedback controller $u = K(\delta\Psi)$ satisfying $K(0) = 0$ and $\delta\Psi = \Psi - \Psi_e$, where Ψ_e is very close to $\Psi_c = \Psi_e(\gamma_c)$ with γ_c is the critical value, such that the pitchfork bifurcation is changed from subcritical into supercritical, thereby stabilizing the bifurcated solution, for which the results in Theorem 2.1 is applicable. Denote

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \Psi - \Psi_e \\ \Phi - \Phi_e \\ A \end{bmatrix}, \quad g(x) = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \frac{\sqrt{\Psi}}{\beta^2}, \quad \Psi = \Psi_e + x_1.$$

The following result could be easily established.

Theorem 2.2 *Suppose that $c_0 + c_1 + c_3 \neq 0$, $c_0 - 2c_3 > 0$, and $c_3 < 0$. Then the Moore-Greitzer model described by (7) - (9) with $u = 0$ exhibits pitchfork bifurcation at $\gamma = \gamma_c = 2/\sqrt{c_0 + c_1 + c_3}$, and the bifurcated solution is subcritical, or unstable if $c_0 + 10c_3 < 0$, and supercritical, or stable if $c_0 + 10c_3 > 0$.*

Suppose that the bifurcated solution is subcritical. Then feedback control law can be employed to eliminate the hysteresis loop. Two feedback control laws were proposed in our program:

$$u_n = K_n \frac{(\Psi - \Psi_c)}{\sqrt{\Psi}} = K_n \left(\frac{x_1}{\sqrt{x_1 + \Psi_c}} \right), \quad u_l = K_l(\Psi - \Psi_c) = K_l x_1, \quad \Psi_c = c_0 + c_1 + c_3,$$

for local stabilization of bifurcated solutions at $\gamma = \gamma_c$. The first one is referred to as the nonlinear feedback controller, and the second one is referred to as the linear controller. Our results were the following:

Theorem 2.3 *There exists a stabilizing feedback controller $\tilde{u} = K_n y$ that makes $\tilde{\lambda}_2^* < 0$, i.e., changes the pitchfork bifurcation from subcritical into supercritical, if and only if the following two inequalities*

$$\frac{12c_3(1 + K_n\Psi_c)}{\Psi_c - (1 + K_n\Psi_c)(c_1 + 3c_3)} + 1 > 0, \quad \beta^{-2}(1 + K_n\Psi_c) > (c_1 + 3c_3)\Psi_c, \tag{11}$$

admit a real solution K_n where $\Psi_c = c_0 + c_1 + c_3 > 0$, $c_0 + 10c_3 < 0$, and $c_3 < 0$.

Theorem 2.4 *There exists a stabilizing feedback controller $u = K_l y$ such that $\tilde{\lambda}_2^* < 0$, i.e., it changes the pitchfork bifurcation from subcritical into supercritical, if and only if the following two inequalities*

$$\frac{12c_3(1 + K_l\Psi_c^{\frac{3}{2}})}{\Psi_c - (1 + K_l\Psi_c^{\frac{3}{2}})(c_1 + 3c_3)} + 1 > 0, \quad \beta^{-2}\left(1 + K_l\Psi_c^{\frac{3}{2}}\right) > (c_1 + 3c_3)\Psi_c, \tag{12}$$

admit a real solution $K_l \neq 0$ where $\Psi_c = c_0 + c_1 + c_3$.

In the above two theorems, the stabilization feedback laws can be characterized explicitly if additional conditions $|c_1 + 3c_3| < \beta^{-1}$, and $c_1 - 9c_3 > 0$ are imposed which are true for the compression system in [5]. In this case the stabilization gains are given by

$$-\frac{1}{c_0 + c_1 + c_3} < K_n < \frac{1}{c_1 - 9c_3} - \frac{1}{c_0 + c_1 + c_3}, \tag{13}$$

$$\frac{1}{\sqrt{c_0 + c_1 + c_3}} \left(\beta^2(c_1 + 3c_3) - \frac{1}{c_0 + c_1 + c_3} \right) < K_l < \frac{1}{\sqrt{c_0 + c_1 + c_3}} \left(\frac{1}{c_1 - 9c_3} - \frac{1}{c_0 + c_1 + c_3} \right). \tag{14}$$

Our results were tested in computer simulations using parameters in the compressor model of Moore-Greitzer [5]:

$$\lambda = 1.75, \quad H = 0.18, \quad W = 0.25, \quad B = 2, \quad a = 1/3.5,$$
$$c_0 = 8/3, \quad c_1 = 1.5, \quad c_3 = -0.5, \quad l_c = 8, \quad l_F = \infty.$$

The simulation results show that the critical operating condition was stabilized and the hysteresis loop associated with rotating stall was eliminated.

(3) Simulation study for various feedback stabilization laws in rotating stall control with high order multi-mode compressor models of Moore-Greitzer.

Because our control law was developed for low order Moore-Greitzer model, its effectiveness for high order model or the full PDE (partial differential equation) model was not known. Hence computer simulation tools were developed in Simulink Toolbox by our ASSERT Program to validate our design. Because many of the other feedback laws in open literature were also developed for the third order Moore-Greitzer model, their effectiveness for high order compressor model was also examined. For this purpose, the "distributed model" in [7] was refined further to yield a multi-mode Moore-Greitzer model. The total order of this model is $2N+2$ with N the number of pairs of critical modes corresponding to rotating stall. This is an approximation of the full PDE Moore-Greitzer model, and in the limit $N \to \infty$, the multi-mode Moore-Greitzer model converges to the full PDE Moore-Greitzer model uniformly. Our results are as follows:

- For the linear and nonlinear feedack control laws in Theorem 2.3 and Theorem 2.4, both stabilized the critial operating condition. However, as throttle position decreased beyond the critical operating point, a secondery bifurcation occurred which happened to be subcritical, inducing hysteresis loop. This secondery bifurcation limited the use of control laws developed in Theorem 2.3 and Theorem 2.4. At present the PI is working closely with his students on modifying these two control laws such that they eliminate the hysteresis loop in the entire operating range (below and above the critical operating condition).

- For the original Liaw and Abed feedback control law, it was generalized into

$$A^2 = \sum_{n=1}^{N} (A_n^2 + B_n^2)$$

  with A_i and B_i the Fourier series coefficients for $\cos(\cdot)$ and $\sin(\cdot)$ respectively. It stabilizes the multi-mode Moore-Greitzer model for $N > 1$, and no secondery bifurcation was discoverd. However as $N \to \infty$, 2-D sensors are required.

- The feedback control law using backstepping method was also examined [2]. It has the form:

$$u = K_A \left[\pi \sum_{n=1}^{N} (1 + \frac{a\lambda}{n}) n^2 (A_n^2 + B_n^2) \right]^{1/2} + K_\Phi(\Phi - \Phi_0) + K_\Psi(\Psi - \Psi_0).$$

Thus it is also a generalization of the Liaw and Abed control law. Similar stabilization result was achieved with the backstepping control law, and no hysteresis loop occurred in the entire operating range. Similarly, 2-D sensors are required as $N \to \infty$.

- The last control law examined was the following:

$$u = K_\Psi(\Psi - \Psi_0) + K_A \sum_{i=1}^{k} (\phi_{n_i} - \Phi_0)^2$$

with n_k fixed, independent of N, and ϕ_i the local flow coefficient at the circumference of the compressor duct. The reason for study such a feedback control law is that it avoids 2-D senors, since number of output measurement for local flow coefficients is fixed, and independent of N. It is very interesting to note that this control law achieves the same stabilization result as the previous two, but the required stabilization gain K_A is greater.

A technical report was written that described the above results in detail. It was made available to the control group at WPAFB (Wright-Patterson Air Force Base). The computer codes in MATLAB are available upon request.

2.2 Design of Missile Autopilots

There had been some new developments in recent years for control of nonlinear systems using gain schedule approach. Various techniques were employed to formalize the gain schedule with the aim to establish unified framework in which both stability and performance of nonlinear feedback control systems can be studied. Notably, Shamma and Athans [12] converted the control of nonlinear systems into that of linear parameter varying (LPV) systems where slow varying parameters or exogenous signals were used as "scheduling variables". A similar approach was also used by Rugh [11] in conjunction with the local linearization but with different "trim conditions". Packard [9] approached gain schedule from different perspective using structured singular value where uncertain parameters were chosen as scheduling variables. A common feature in [9, 11, 12] and their corresponding applications to pitch autopilot design was that robust control of linear systems such as $\mathcal{H}_\infty$ and μ synthesis were employed to design a set of linear feedback controllers and each ensures stability and performance of the feedback system on a range of operating conditions or parameter variations. In our research program, a different approach was taken. In stead of gain schedule method, a time-varying control design method was proposed to the design of missile autopilots. The spectrum theory for time-varying systems was employed that yielded better performance and simpler design procedures. In the following, our results will be described briefly.

(1) Design of a missile autopilot for angle of attack and nominal acceleration tracking using extended-mean (EMA) control technique for time-varying systems.

The autopilot was to control the nonlinear time-varying pitch-axis dynamics of a hypothetical tail-controlled missile, which had been used as a benchmark in a number of recent studies on nonlinear gain-scheduling design techniques. The nonlinear pitch dynamics of the missile was rendered into an LTV system via classical linearization along a nominal trajectory, and then operated on by a linear coordinated transformation to make it tractable by the EMA control technique. Our departure from the conventional design philosophy was that nonlinearity and time variance are not treated as nuisances, but purposely utilized to accomplish design objectives beyond the capability of LTI controllers. Salient features of the EMA tracking controllers include: (a) Good tracking performance for arbitrary trajectories without scheduling of any constant design parameters throughput the entire Mach operating range; (b) Time-varying EMA command, or pole locations to improve tracking performance; (c) Implementation of the inverse pitch dynamics using static neural network, and (d) a time-varying bandwidth command shaping filter that effectively reduces the actuator rate while maintaining good tracking response for both smooth and abrupt trajectories. Our simulation results demontstrated the excellent tracking for angle-of-attack for step, and sinusoidal trajectories, with constant EMA commands and with nominal and $\pm 50\%$ variations in the aerodynamic coeffficents, and normal acceleration tracking for both step and sinusodal trajectories using an angle-of-attack observer. This was shown by shorter rise time, settling time, and smaller overshot compared with other existing design method.

(2) Design of angle of attack tracking autopilots for a tail fin controlled pitch missile using PD-spectrum assignment controller.

The motivation for our research here is the inherent nonlinearity and time variance for the high performance missile autopilot design which involve large inherent nonlinearities as well as many exogenous states leading to a considerable time dependence. As such, the currently well developed linear time-invariant control theory is of limit utility. The parallel differential (PD) spectral theory was used in the design of angle of attack tracking autopilots for a tail fin controlled pitch missile. It is noted that linearization of the plant about a nominal trajectory results in a linear time-varying (LTV) tracking error model. PD-spectrum assignment guarantees exponential stability of a LTV system. Our design procedure for the underlying missile autopilots consisted of two separate components. The first was a neural network which implemeted a pseudo-inversion of the given plant so as to place the states on the nominal trajectory. The second design component was a PD-spectrum assignment controller which exponentially stabilized the linearized error dynamics about the desired trajectory. It

is noted that if the linearized error dynamics is exponentially stable, then the NLTV plant is locally exponentially stable about the nominal trajectory. Owing to the exponential stability, bounded disturbances only cause bounded changes in output. Our design procedure relied on a Lyapunov transformation of the linearized dynamics applicable to any nth order, uniformly controllable system. The simulation results validated the proposed design method whose performance is comparable to other design methods, and illuminated some of the special inherent strength of this method.

(3) Synthesis of a class of time-varying bandwidth (TVB) filters using a PD-spectrum concepts for time-varying systems.

Our research work on design of missile autopilots using time-varying spectrum theory motivated us to study time-varying filters. It had been long desired by control and signal processing engineers to be able to change controller/filter parameters in real time in the presence of nonlinear, time-varying dynamics and nonstationary signals. Our work was particularly motivated by the need for a guidance tracking command shaping filter for a missile pitch autopilot with actuator displacement and rate limiters. It was desired to have a filter that has a sufficient bandwidth to pass slowly varying commands with little phase lag, yet effectively limits the actuator rate for abrupt trajectory commands by narrowing its bandwidth in real time. These two requirements could not be achieved with a linear time-inariant filter. Our results were obtained based on PD-eigenvalue theory for LTV systems. A synthesis procedure was proposed to design a class of all-pole low pass TVB filters. Stability was guaranteed by correctly synthesizing time varying filter parameters according to the PD-eigenvalue stability criterion. Time-varying frequency response of the TVB filter was also examined in thelight of Zadeh's system function, and a computableprocedure was used to obtain "time-varying Bode plots". The results were again validated by simulation examples.

3 Personnel Supported

This research grant supported two students: Phillip Martin (M.S.), and Michael Mickle (Ph.D.). The details are given as follows.

- Graduate Research Assistant: Phillip Martin.

  Phillip Martin was initially supported as a student worker in 1995 when he was an undergraduate student. In 1996, he was enrolled as a graduate student working with the PI in the area of compressor control. He took the special topics class of the PI on bifurcation stabilization and compressor control, and subsequently became a research assistant of the PI. He finished his Master degree in December of 1997, and is currently working with the Air Transport Systems

of Honeywell Inc. His working experience with the PI on compressor control was considered an asset by the Honeywell.

- Graduate Research Assistant: Michael Mickle.

  Michael Mickle became the research assistant in this ASSERT Program since 1997. He had been interested in missile autopilot design, and was a Ph.D student in the ECE Department, LSU for more than three years. He took quite a number of control courses from the PI at the garduate level, and was well qualified to undertake the research work in missile autopilot design. The first version of his dissertation (focusing on missile autopilot design) was completed. He is currently supported by teaching assistant, and is scheduled to graduate in December of 1998.

4 Publications

Our ASSERT program in the past three years produced 2 journals publications, and 3 conference papers, including those accepted for publications. These papers are listed as follows:

1. X. Chen, G. Gu, P. Martin, and K. Zhou, "Rotating stall control via bifurcation stabilization," *Automatica*, vol 34, 437-443, April 1998; Also presented in *Proc. of IEEE Symp. Cir. and Syst.*, June 1998.

2. J. Zhu and M.C. Mickle, "Missile autopilot design using a new linear time-varying control technique," *J. of Guidance, Control, and Dynamics*, vol. 20, 1-8, Jan. 1997.

3. M.C. Mickle and J. Zhu, "Nonlinear missile planar autopilot design based on PD-spectrum assignment," *Proc. of 36th IEEE Conf. Dec. and Contr.*, 3914-3919, Dec. 1997.

4. J. Zhu and M.C. Mickle, "Synthesis of time-varying bandwidth filters based on all-pole LTI prototypes," accepted by *37th IEEE Conf. Dec. and Contr.*, 1998.

In addition to the above publications, three technical reports were written by Phillip Martin (one of them co-authored by Calin Belt and the PI):

1. P. Martin, C. Belta and G. Gu, "Simulation study for axial flow compressors," Dept. of ECE, LSU, 1997.

2. P. Martin, "Design of experimental compressor," Dept. of ECE, LSU, 1997.

3. P. Martin, "Experimental set up and manual for rotational inverted pendulum," Dept. of ECE, LSU, 1996.

5 Interactions/Transitions and Others

a. The PI had given numerous presentations in major control conferences, and been invited to give several seminars in different research institutes. Because conference presentations can be found in the publication section, only those invited seminars in the past three years are listed below:

(1) G. Gu, "Bifurcation stabilization and compressor control," College of Engineering, University of California at Riverside, June 1997.

(2) G. Gu, "Bifurcation stabilization and applications to rotating stall control for axial flow compressors," Department of Mathematics, Naval Post Graduate School, Monterey, CA, July 1996.

(3) G. Gu, "Modeling of coprime factors with gap metric uncertainty," Wright-Patterson AFB, Dayton, Ohio, July 1995.

Two students also attended conferences, and gave presentations. See the conference papers in the publication list.

b. In the past year, the PI collaborated closely with the control group of Dr. Siva Banda in WPAFB. Two groups visited each other at least twice a year, and had several joint papers in conferences and technical journals. The PI is currently collaborating with Dr. Jim Paduano at MIT working on identification of acoustic modes for high speed compression systems which will be an important move for us to enhance the application part of our research program.

c. Technology Transitions.

Performer:	Professor Guoxiang Gu, Louisiana State University.
Telephone:	(504) 388-5534.
Customer:	Compression System Component Center, Pratt & Whitney
Contact:	Dr. Carl N. Nett, Director
Telephone:	(937) 255-8682
Result:	Bifurcation stabilization; Active control of rotating stall and surge for Moore-Greitzer model to enlarge stable operating range.
Application:	Design of active feedback control laws to suppress rotating stall and surge, and to improve the performance of axial flow compressors.

d. New discoveries, inventions, or patent disclosures: None.

e. Honors/Awards: None.

6 Conclusion

During the past three years, our research program had accomplished what was initially planned. Due to the change of research objectives for the parent proposal, compressor control became an important part of this program, and missile autopilot design remained as the other focus of our research work. During the three year period, this program supported one M.S. student, and one Ph.D student. The research results of these two students were reported in two journal papers, three conference papers, and three technical reports. The significance of their results was shown in the rotating stall control laws for axial flow compressors, and new design tools for missile autopilots that improve the tracking performance of the flight control systems. Hence both of our research and educational objectives were accomplished. In addition, the PI had offered two special topics courses at the graduate level. More than twenty graduate students were benefited through the course work and class projects in these two classes. These educational activities helped greatly train students to participate the research work in both the parent proposal and this program. Our program was very fortunate to work under the guidance of Dr. Jacobs during our three year period, and to have a strong tie with the control group in Wright Laboratory at Wright-Patterson Air Force Base led by Dr. Siva Banda. With the preparation of this research program and close collaboration with the control group at Wright-Patterson Air Force Base led by Dr. Siva Banda, our research team is well positioned to undertake new research tasks in the interest of AFOSR. Our control group at LSU is confident that our research program has the capability to contribute further to the DoD mission in the near future.

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